

Probabilistic Modeling of Earthquake Occurrence:

First examples of data integration within a Bayesian framework

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Motivation

- Why and how build physics-based earthquake simulators?
- Can we use lab experiments to constrain large-scale fault models? what are the limitations?
- What is the best way to infer microphysical processes from macroscopic measurements?
- How can we account for observational errors, model uncertainties, and potential deformation regime changes during each experiment?

Renewal models used in Seismic Hazard Assessment

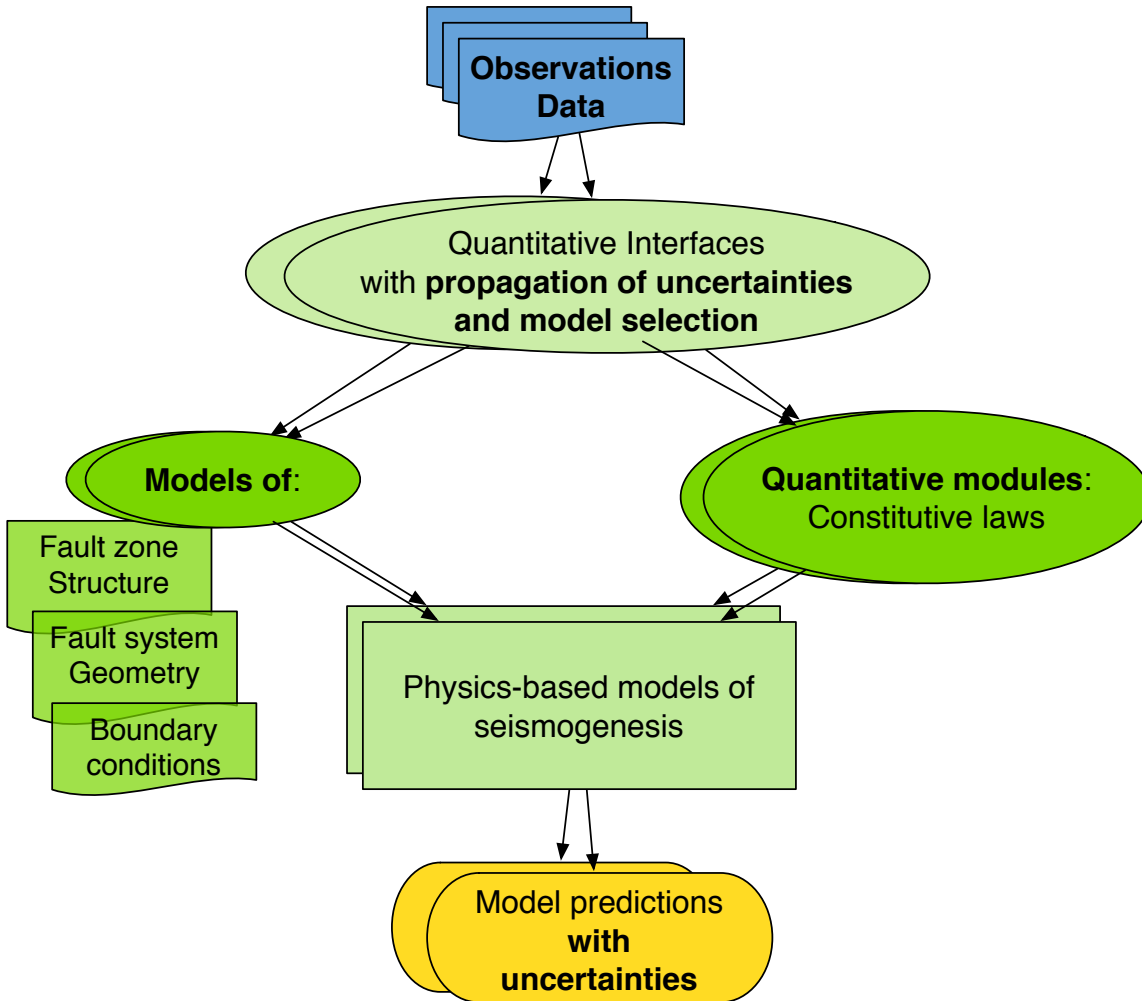
The four rupture probability models to be applied to each fault currently proposed in OpenSHA are:

- the time-independent Empirical Poisson,
- the Brownian Passage Time (BPT) model, which depends on the time since the last event (Matthews&Ellsworth, 2002),
- the BPT-step model, which accounts for earthquake-induced stress changes,
- the time-predictable model.

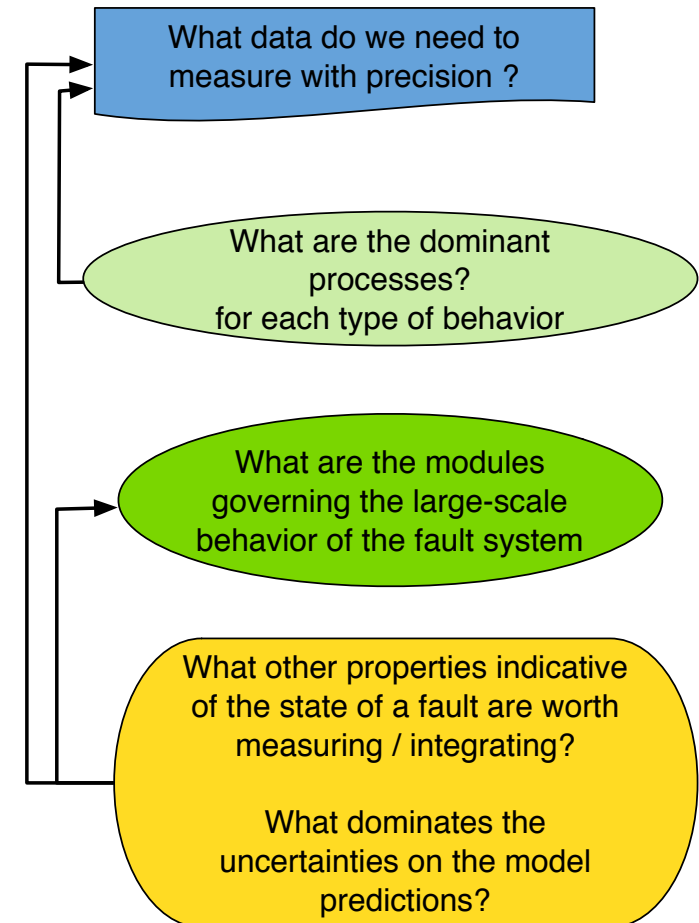
No Fault Physics

Motivation for Bayesian, physics-based, models of seismogenesis

BAYESIAN MODELLING FRAMEWORK FOR DATA ASSIMILATION



EXPECTED RESULTS

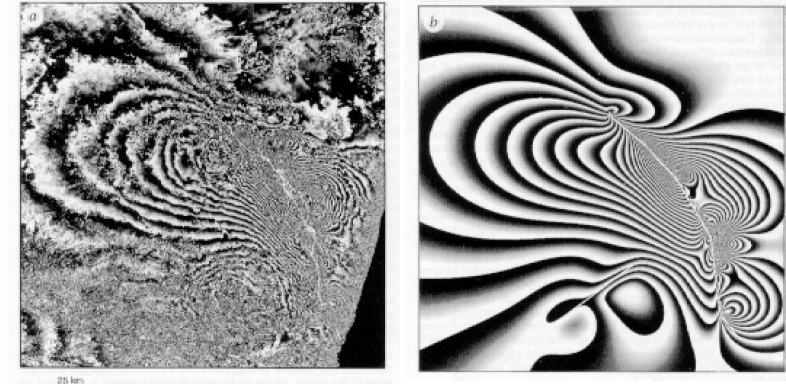


After Fitzenz et al. Statseis V, 2007

Traditional bases of models of sismogenesis: observations and geological or geophysical measurements

1) Elasticity of the upper crust

Landers 1992
Massonnet et al. 93



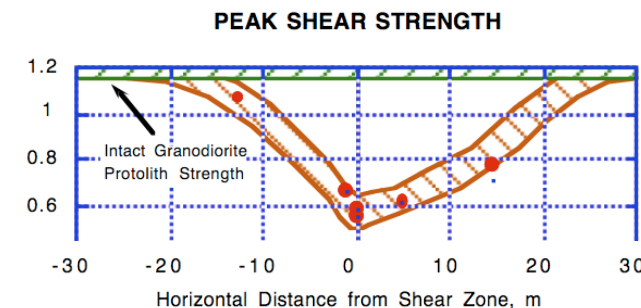
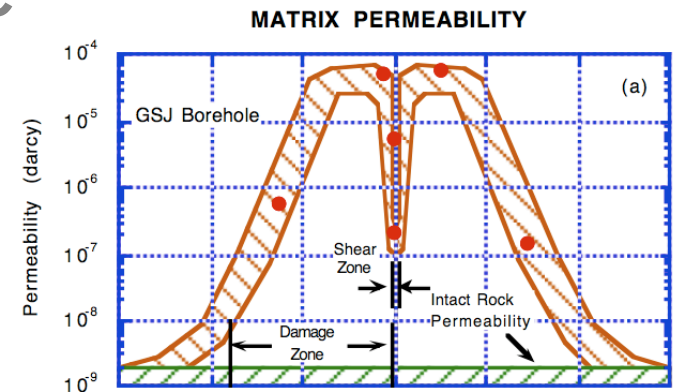
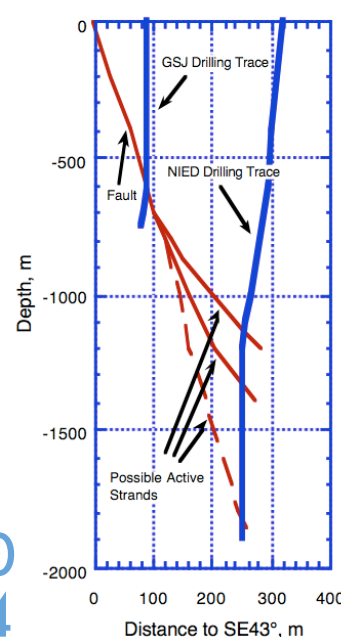
2) Deformation highly localized in a zone

that is

- impermeable
- weak
- place of episodic fluid circulation

Nojima Fault
Lockner et al. 00
Boullier et al. 04

DRILLHOLE/FAULT GEOMETRY

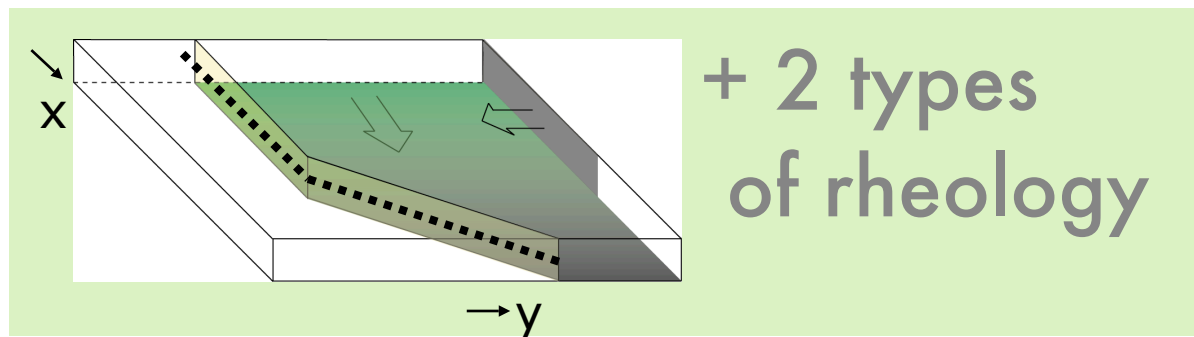


Fault strength and stress orientations

Application to Big Bend in the San Andreas

30 years of debate...

→ Is the stress field stable in time?



→ Close to active faults, local rotations

Need for a monitoring of the space and time evolution of the anomalies in stress orientation

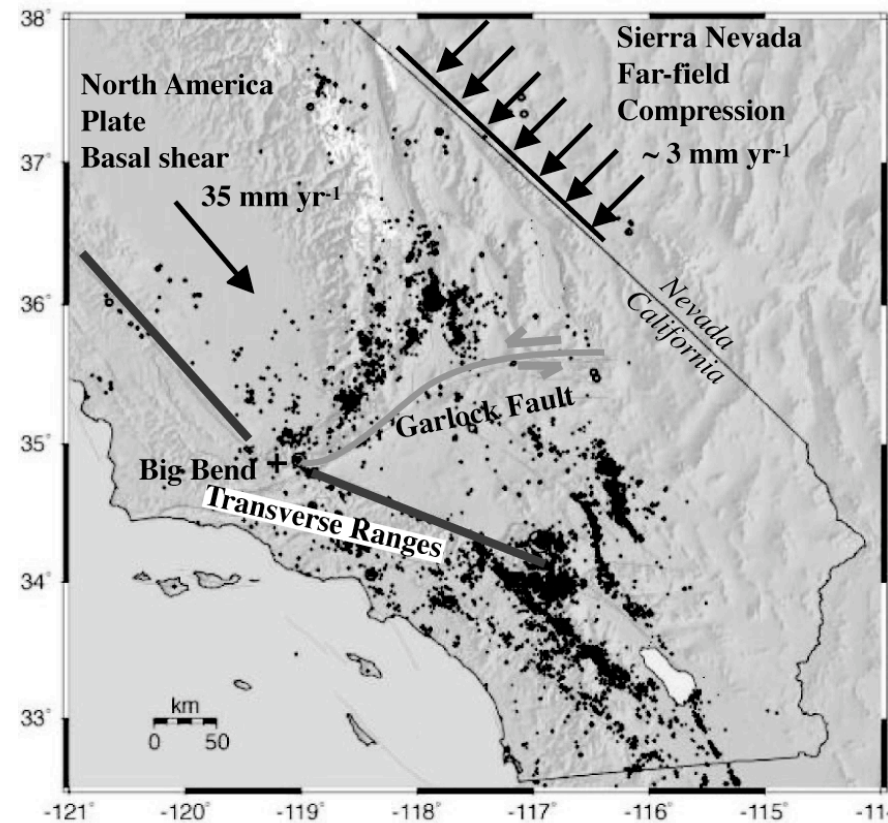
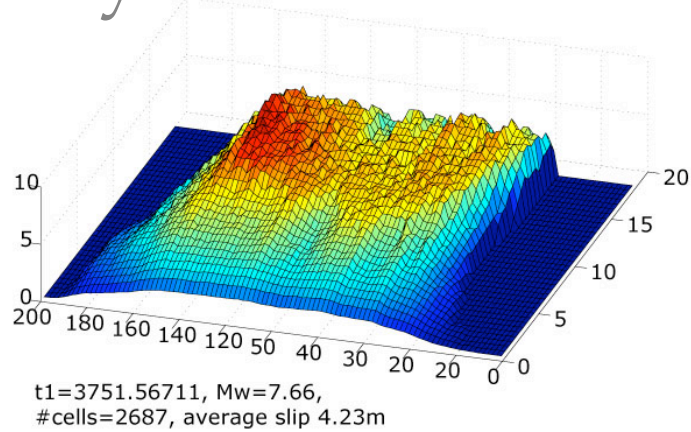


Figure 1. Earthquakes recorded by SCSN/TriNet.

Examples of model results

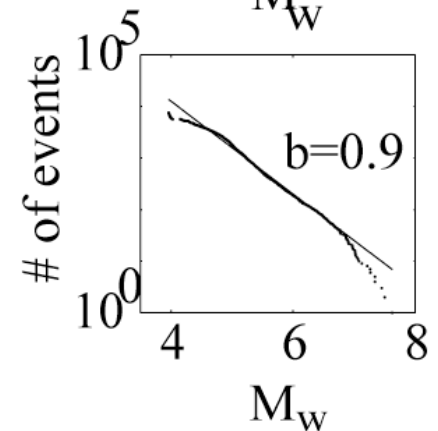
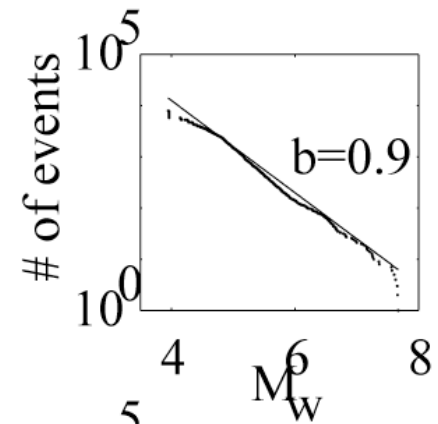
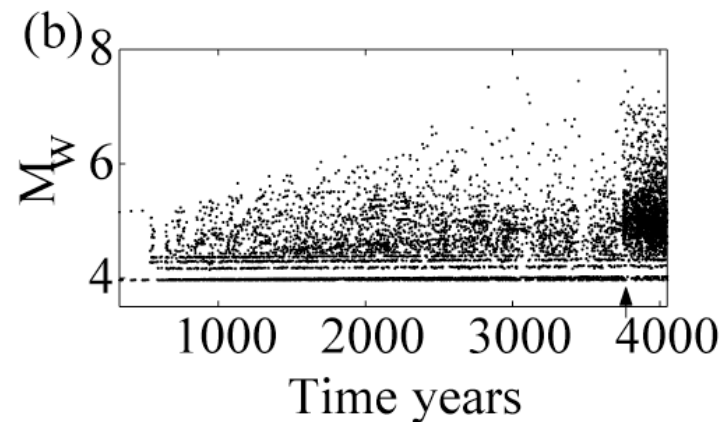
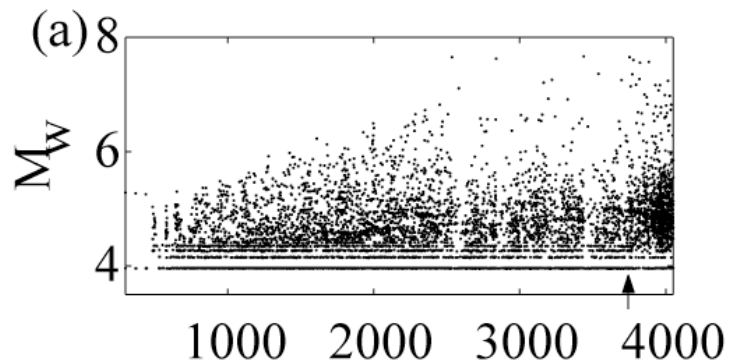
- Synthetic seismicity



Northern Segment

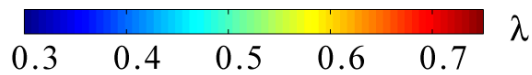
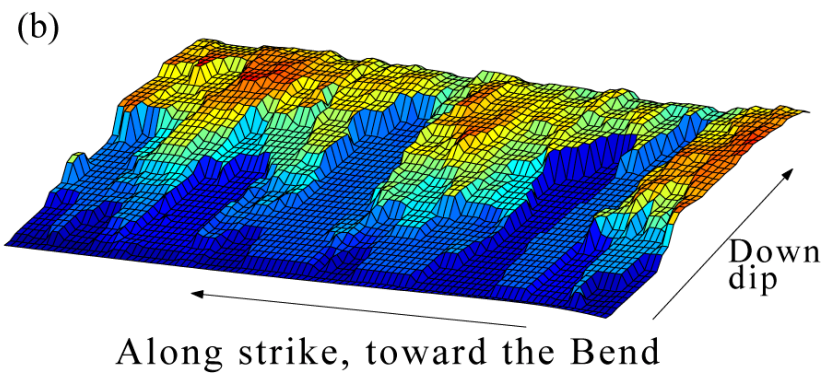
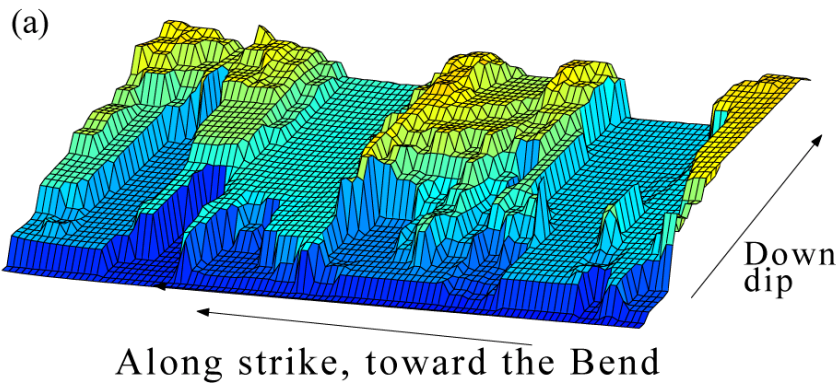
Southern segment

$$M_0 = G \cdot \sum_i \delta_i A_i \quad M_w = 2/3 \log_{10} M_0 - 10.7$$

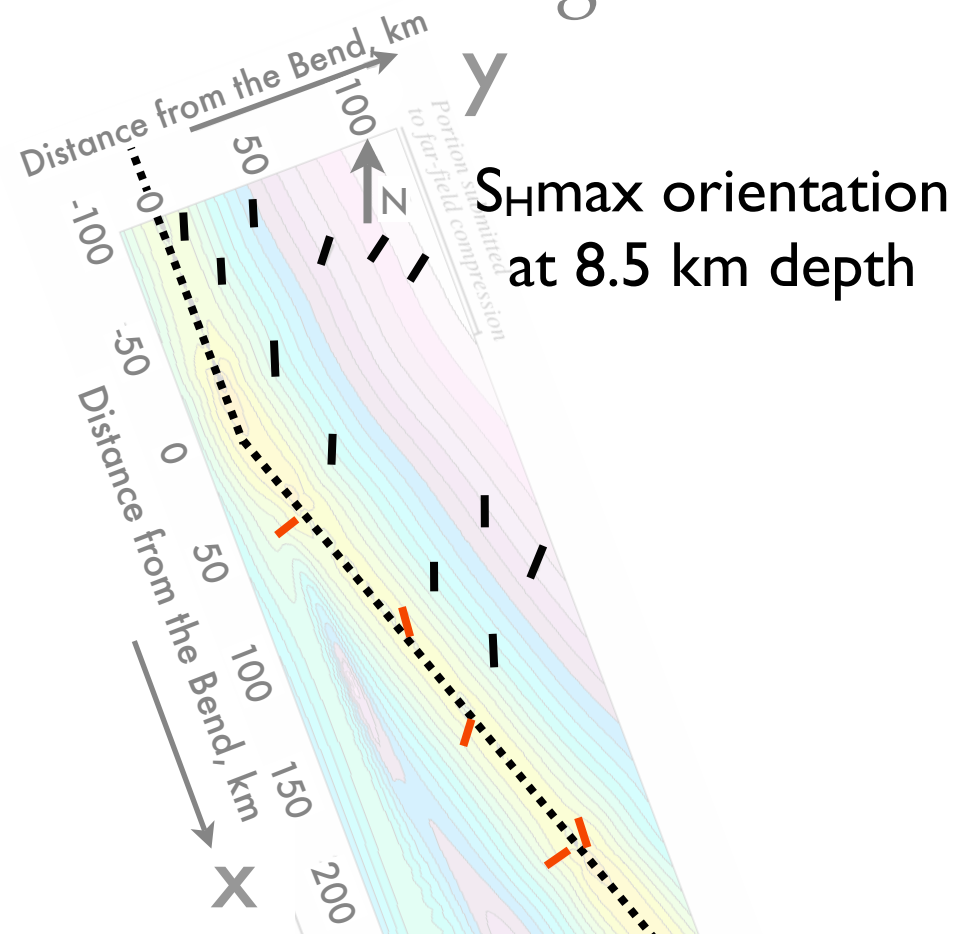


Additional model results

Pore pressure monitoring



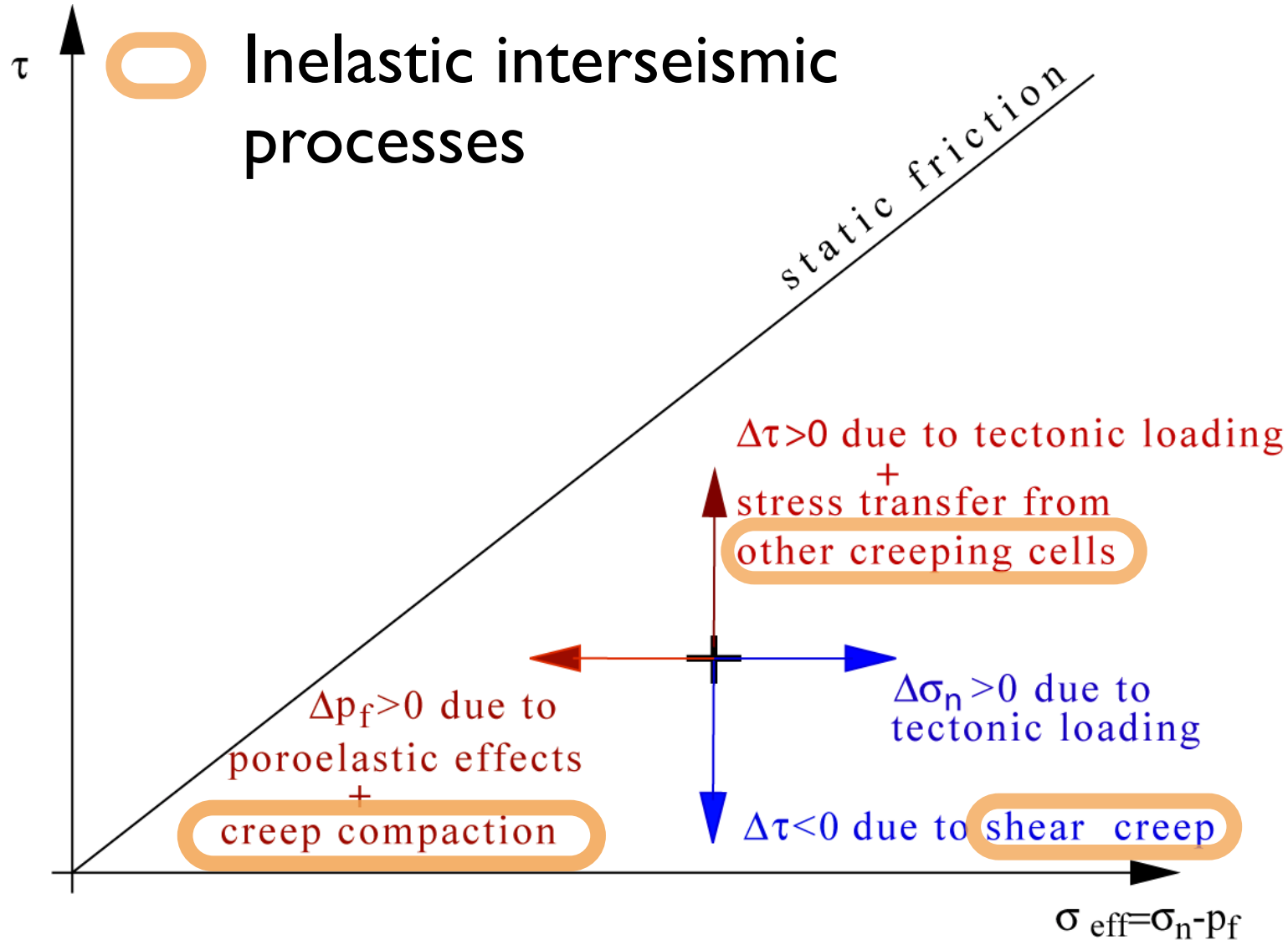
Regional stress state and corresponding deformation regimes



More data integration

- Loading rates (from GPS or such)
 - Fault geometry (from seismicity, seismic or potential fields surveys)
 - Topography (levelling surveys, lidar, optical images)
- But also data from the lab or other sources to constrain the constitutive laws

The role of fluids in faulting



Our Bayesian inference scheme

Observations

Experimental or simulated porosity time series

Choice of the model m



When the grain-size effects can be neglected, and we make no assumption on the mechanisms, we can choose a creep law of the type:

$$\frac{\partial \varphi}{\partial t} = \theta_0 \times \sigma_{\text{eff}}^{\theta_1} \times \exp(-\theta_2/(RT)) \times \exp(\theta_3 \varphi)$$

Stress
exponent

Apparent
activation
energy

Prerequisite

- Integration of the creep law: we want to use the data, not a subproduct of it

- Reparametrization such that the problem is as linear as possible
(work with Gaussians)

The five steps of the inference

A: Choose the parametrization making the problem as linear as possible

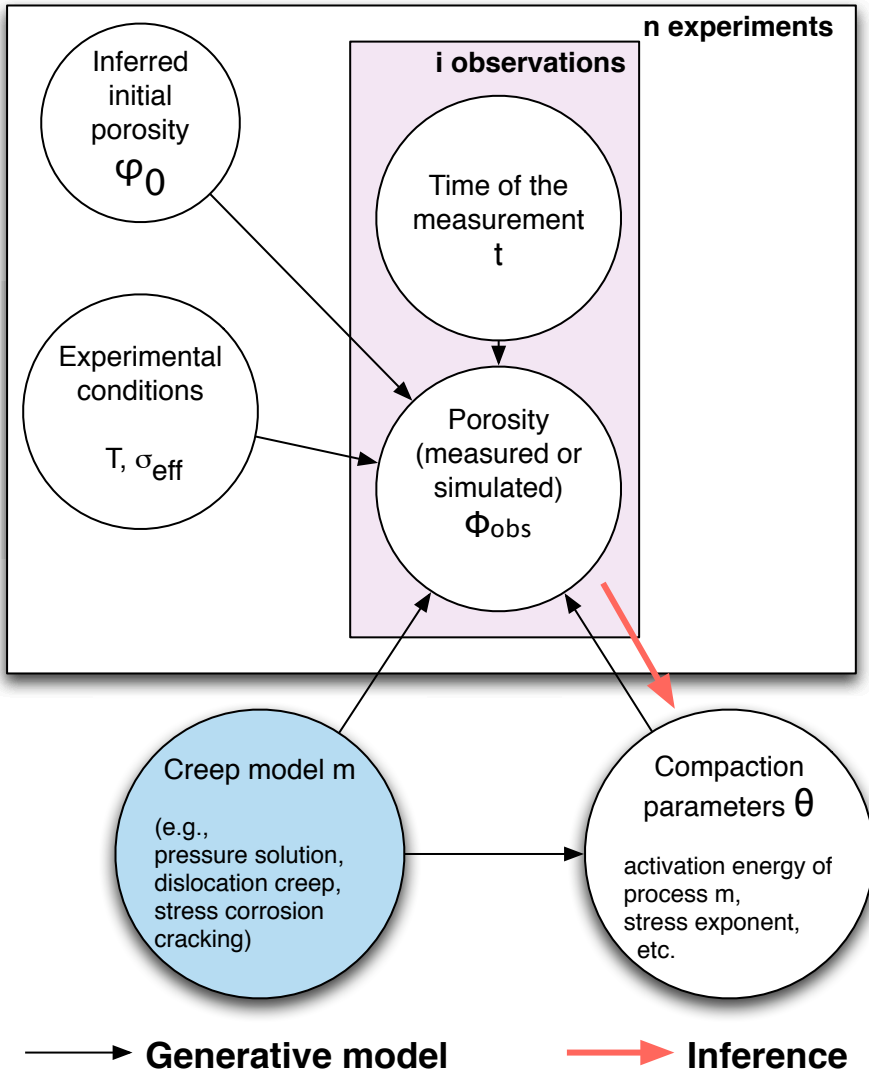
B: When a factorization appears in the new Bayesian network, adopt a hierarchical inference scheme

C: Calculate the joint probability density function according to the graph structure (the joint pdf is proportional to the posterior pdf).

D: Eliminate the nuisance parameters (marginalization step)

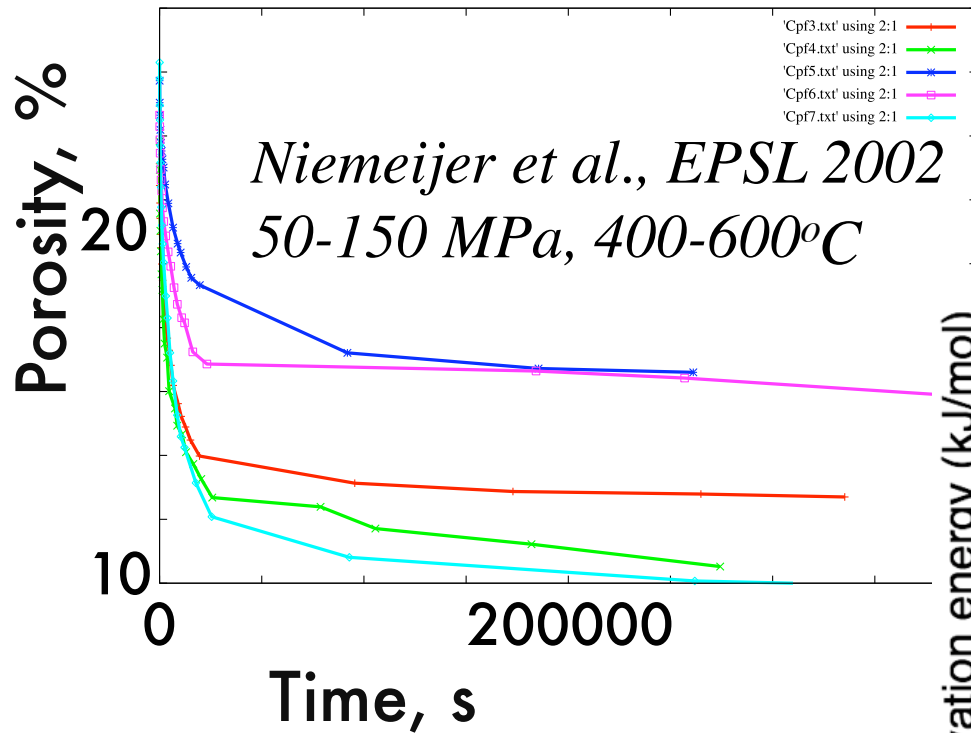
E: Revert from the new parametrization to the original parameters θ_i

Bayesian networks

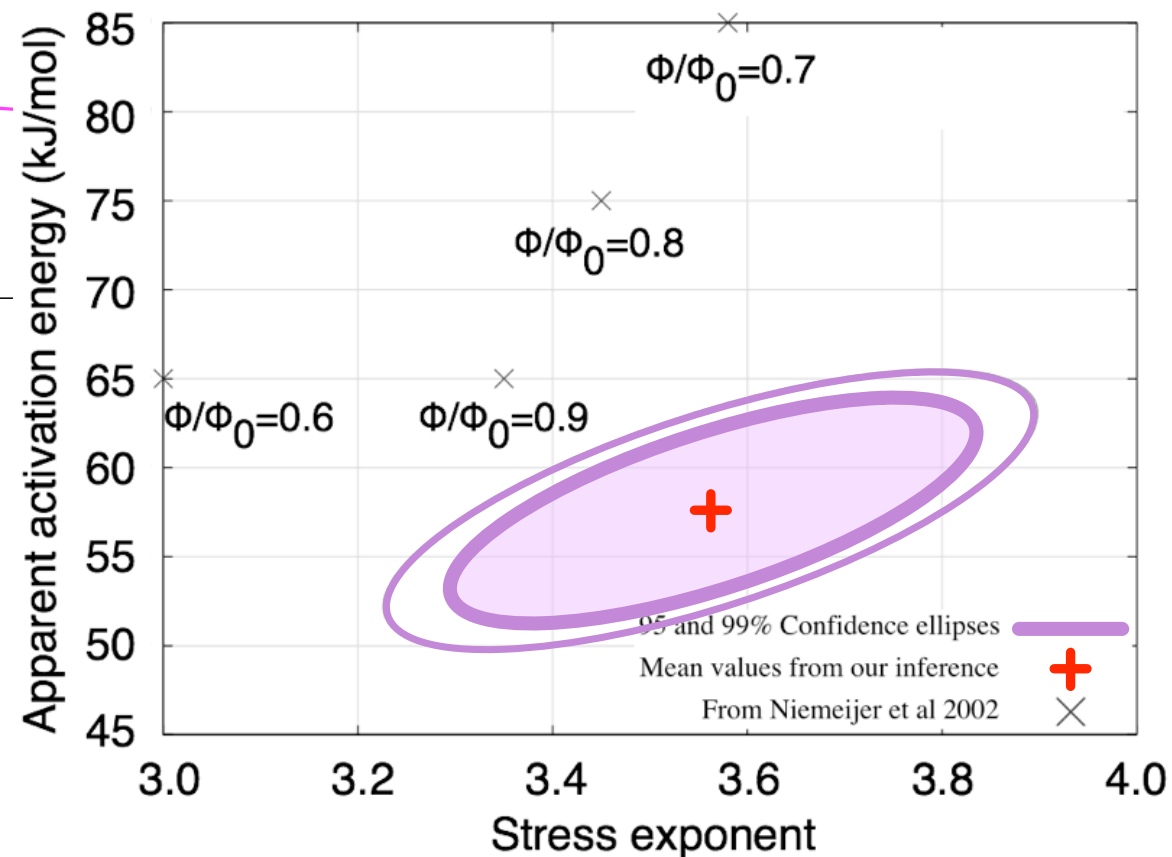


- Each node is a random variable
- Each arrow shows a causality relationship

Application to real experimental creep data



Subset $\Phi/\Phi_0 \in [0.5, 0.8]$: 53 points
 $\theta_j = \{4.2 \cdot 10^{-13}; 3.56; 57.6 \text{ kJ/mol}; 0.71\}$,
 with std. dev. of $2.3 \cdot 10^{-13}$, 0.11, 2.6 kJ/mol,
 and 0.016 resp.



Interpretation and questioning

Stress exponent 3.5

Apparent activation energy 60 kJ/mol

Most likely mechanisms :
compaction rate-controlled by dissolution
+ cataclasis or stress corrosion

However:

The relation between contact stress and applied stress may be affected by changes in grain packing

We need to perform a test on a more controlled system: could we identify pure pressure solution?

Rock physics detour

Validation procedure: simulated pressure solution

Single contact model by Bernabe and Evans 2007
based on the evolution of the contact shape

- Stress exponent 1
- Apparent activation energy:
between 72 kJ/mol (contact dissolution)
and 15 kJ/mol (interface diffusion)
throughout the simulated experiments

Data transformed into porosity time series by assuming
a simple cubic packing geometry + Gaussian noise

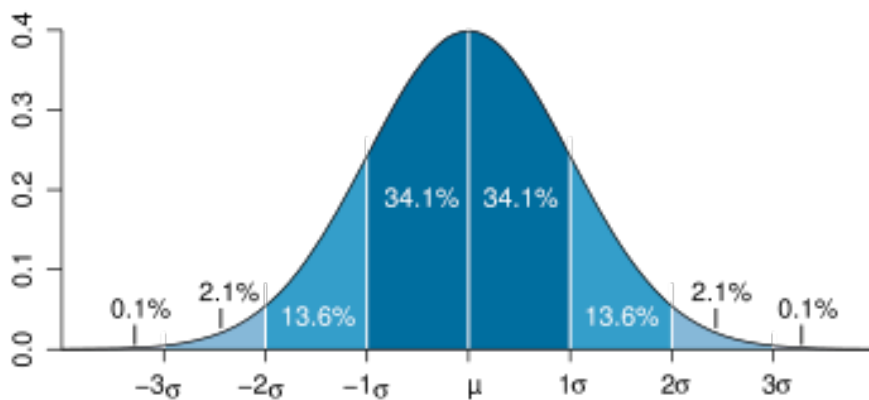
Challenge:

Analyze data whose behavior is driven by the contact area
without knowing how contact area evolves through time

Identifying transitions

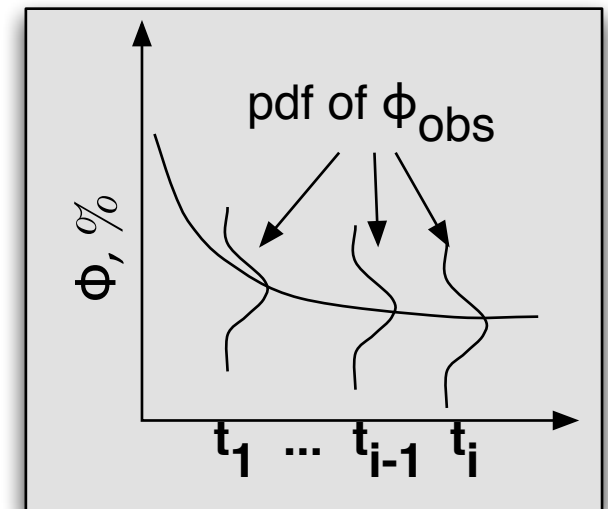
For each experiment, we estimate the maximum number of points n belonging to the same deformation regime using a χ^2 test

x = realization of a random variable
hypothesis testing: x is normally distributed,
variance σ^2 , mean μ
 $|x - \mu| > 2\sigma$: less than 5% likely to have occurred by chance



$$x = \frac{\sum_{n < N} (\Phi - \Phi_{\text{obs}})^2}{\text{noise variance}}$$

A scatter plot showing a series of data points (green crosses) that follow a downward linear trend. A solid red line is drawn through the points, representing a linear fit or trend line.



with $\Phi(t) = m(T, \sigma_{\text{eff}}, \theta, \varphi_0, t)$

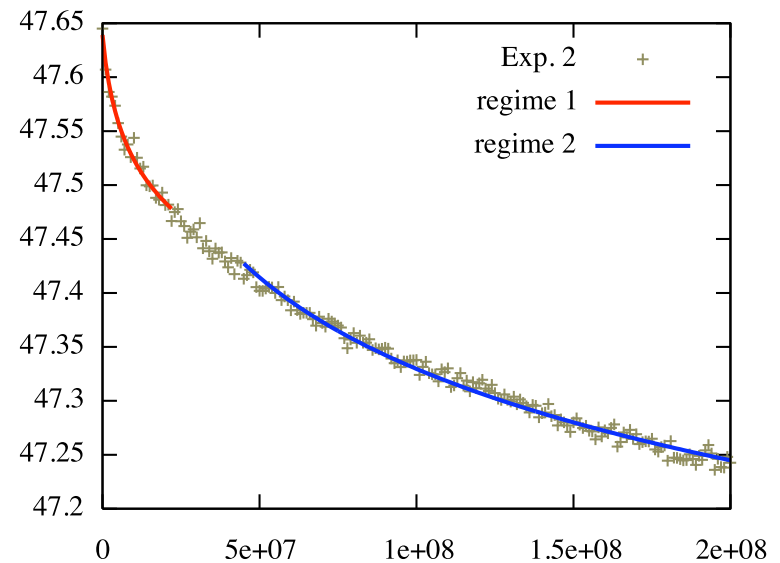
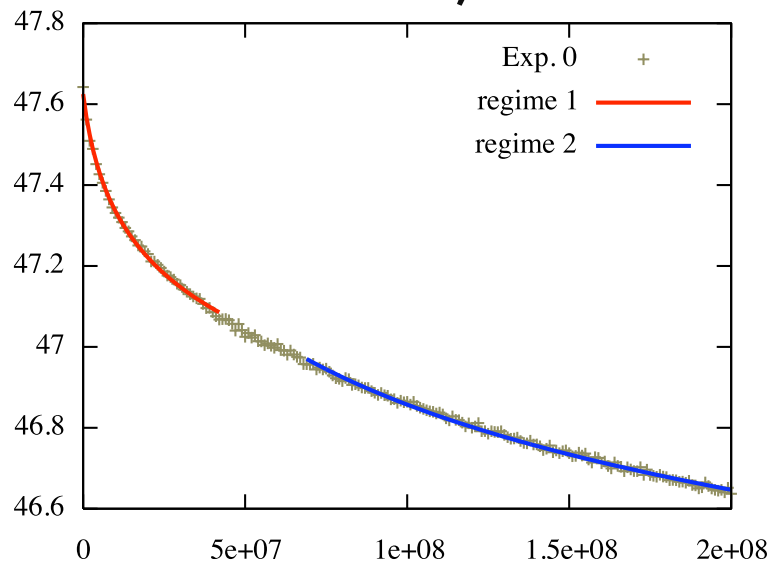
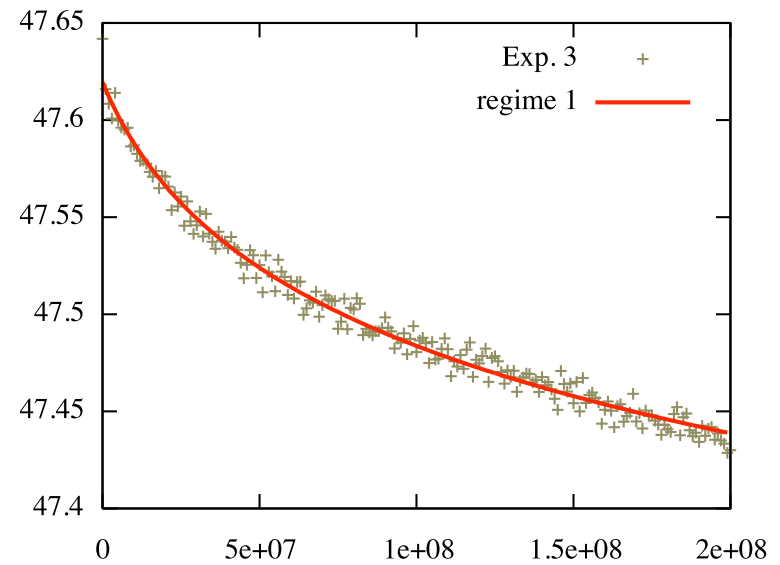
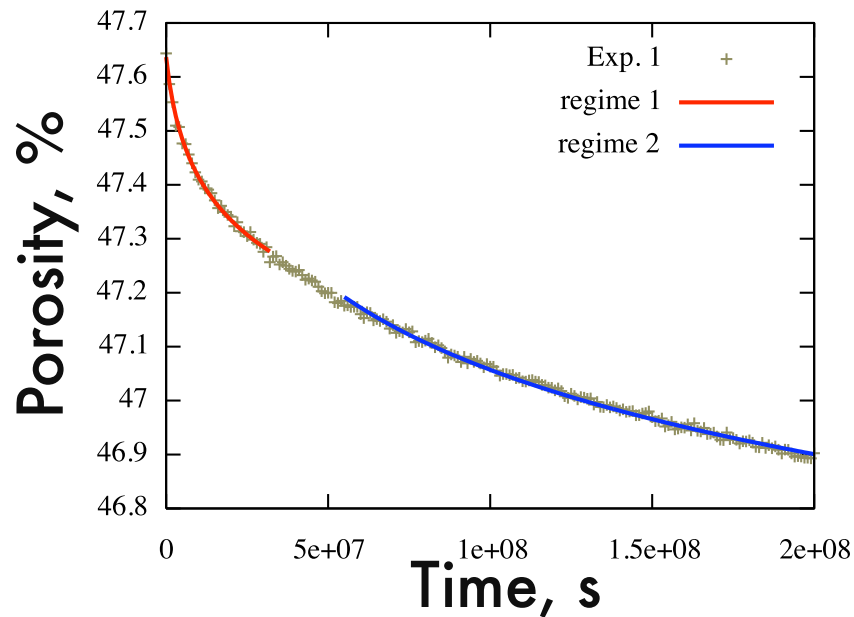
10 simulated experiments

Exp #	σ_n (MPa)	T (K)
0	5.482	508
1	2.564	508
2	0.495	508
3	0.080	508
4	$1.591 \cdot 10^{-2}$	508
5	0.495	574
6	0.495	542
7	0.495	508
8	0.495	472
9	0.495	435

After Bernabe & Evans
2007

201 points/experiment,
Gaussian noise with std dev 0.005

Identifying transitions



The 2 deformation regimes (at 3σ)

Regime 1: Stress exponent $0.97 \pm 4.4 \times 10^{-3}$
Apparent act. energy 56.9 kJ/mol
 $\pm 0.3 \text{ kJ/mol}$

Regime 2: Stress exponent $1.2 \pm 5.4 \times 10^{-3}$
Apparent act. energy 32.8 kJ/mol
 $\pm 0.1 \text{ kJ/mol}$

- Stress exponents close to 1 as assumed in Bernabe & Evans
- Both activation energies between 72 kJ/mol (contact dissolution) and 15 kJ/mol (interface diffusion);
- For the early stage, Bernabe & Evans obtained 58 kJ/mol with a different method (knowing the contact area as a function of time);
- The late times are better approached with our method.

Validation procedure: simulated pressure solution

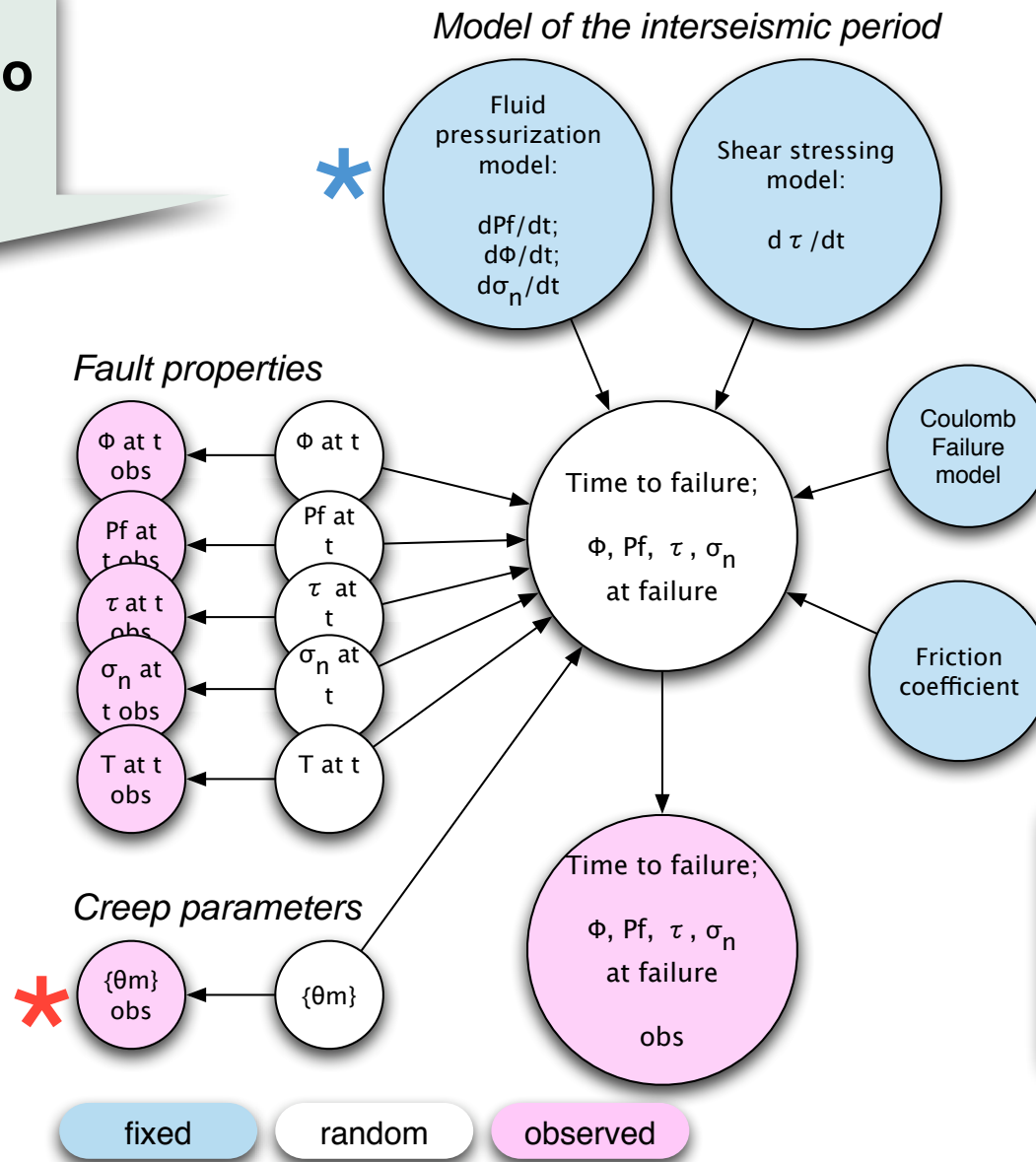
Conclusion:

- With stress exponents close to 1 and a decreasing apparent activation energy from above 57 kJ/mol down to below 33 kJ/mol, we can infer that the deformation is pressure solution, mostly controlled by dissolution at the beginning, and with a larger contribution of interface diffusion as contacts grow.
- The times (and porosities) of the transition are T - and σ_n -dependent.

Back to earthquake
simulators
and seismic hazard
assessment

Time to failure using the creep law inferred using the Niemeijer et al. data

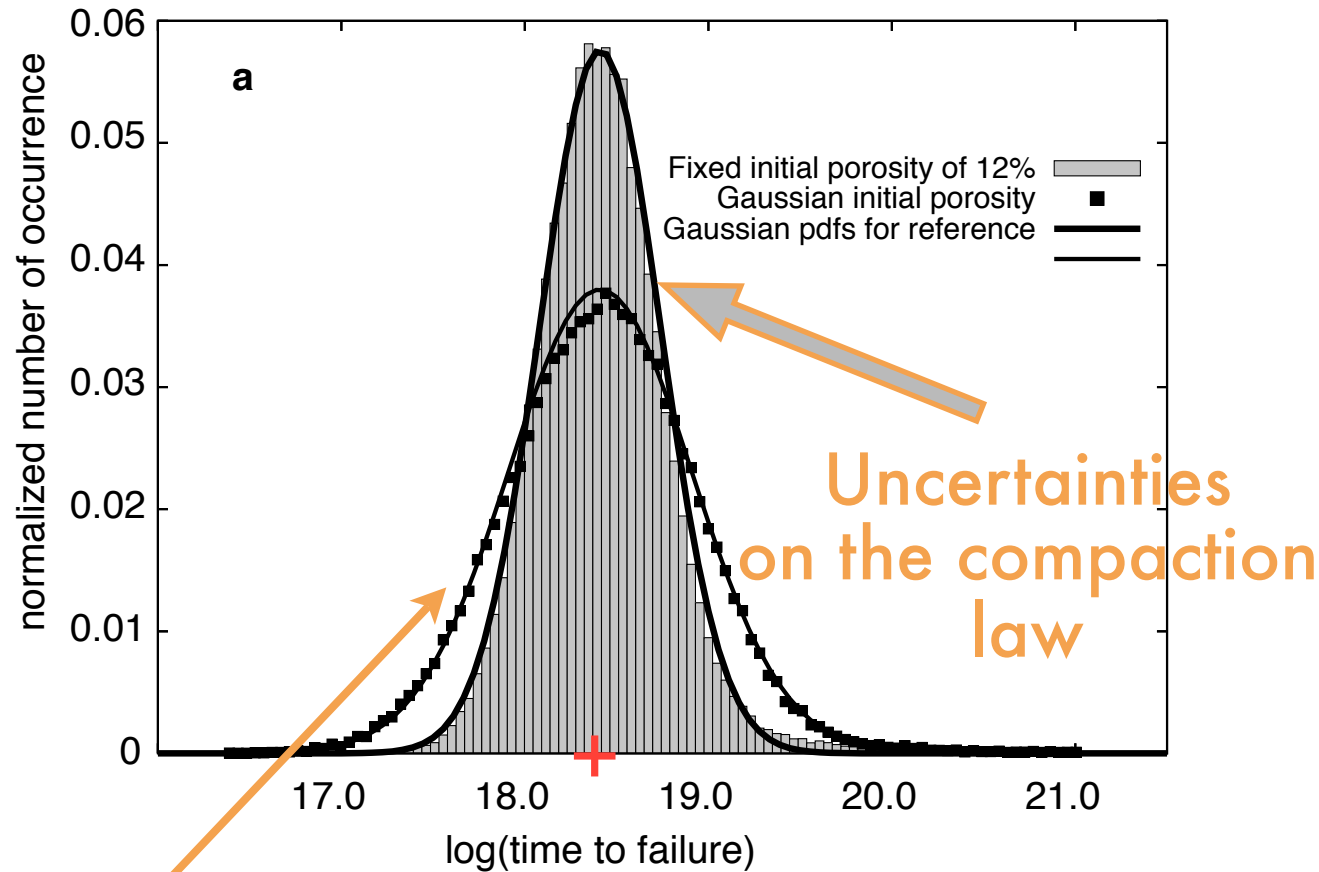
Monte Carlo sampling through



Results in terms of time to failure

Time to failure distributions

- undrained
- 1 fault element
- $t=0$, 12% porosity
- $z=3\text{km}$
- $\{\theta_j\}$ from previous inversion
- (static) friction=0.6
- $d\tau/dt=2.5$ bar/year
- $d\sigma_n/dt=0$
- Coulomb failure:
 $\tau \geq \text{friction} * (\sigma_n - P_f)$



+ Uncertainties on porosity after an earthquake

Conclusions

1. We developed a Bayesian inference scheme that we validated using simulated pressure solution experiments;
2. The analysis of simulated experiments can also guide the design of future experiments;
3. We applied this method to real compaction data obtained at hydrothermal conditions;
4. We showed how to propagate the uncertainties to time to failure distribution;

and future directions

This type of Bayesian framework seems promising for future efforts to compute earthquake probability models including the “known” fundamental physics, different types of data, their observational errors, and the model uncertainties when they exist.

Thank you

The 3 steps of the inference

C A: For each experiment, infer λ' and ν

$$P(\varphi_0, \{\phi^i, \phi_{\text{obs}}^i\}, \lambda', \nu) = P(\varphi_0) \prod_i \underbrace{P(\phi_{\text{obs}}^i | \phi^i)}_{\text{Gaussian pdf}} \underbrace{P(\phi^i | \varphi^i, \lambda', \nu)}_{\text{Dirac}} \delta(f(\varphi_0, t^i, \lambda', \nu) - \phi^i)$$

Joint pdf Prior pdf

D B: Infer Θ from $\{\lambda'^n, \nu^n\}$

$$P(\Theta, \{\lambda'^n, \nu^n, \lambda'_{\text{obs}}^n, \nu_{\text{obs}}^n\}) = P(\Theta) \prod_n \underbrace{P(\lambda'^n, \nu^n | \Theta, m^n)}_{\text{Dirac}} \underbrace{P(\lambda'_{\text{obs}}^n, \nu_{\text{obs}}^n | \lambda'^n, \nu^n)}_{\text{Gaussian pdf}}$$

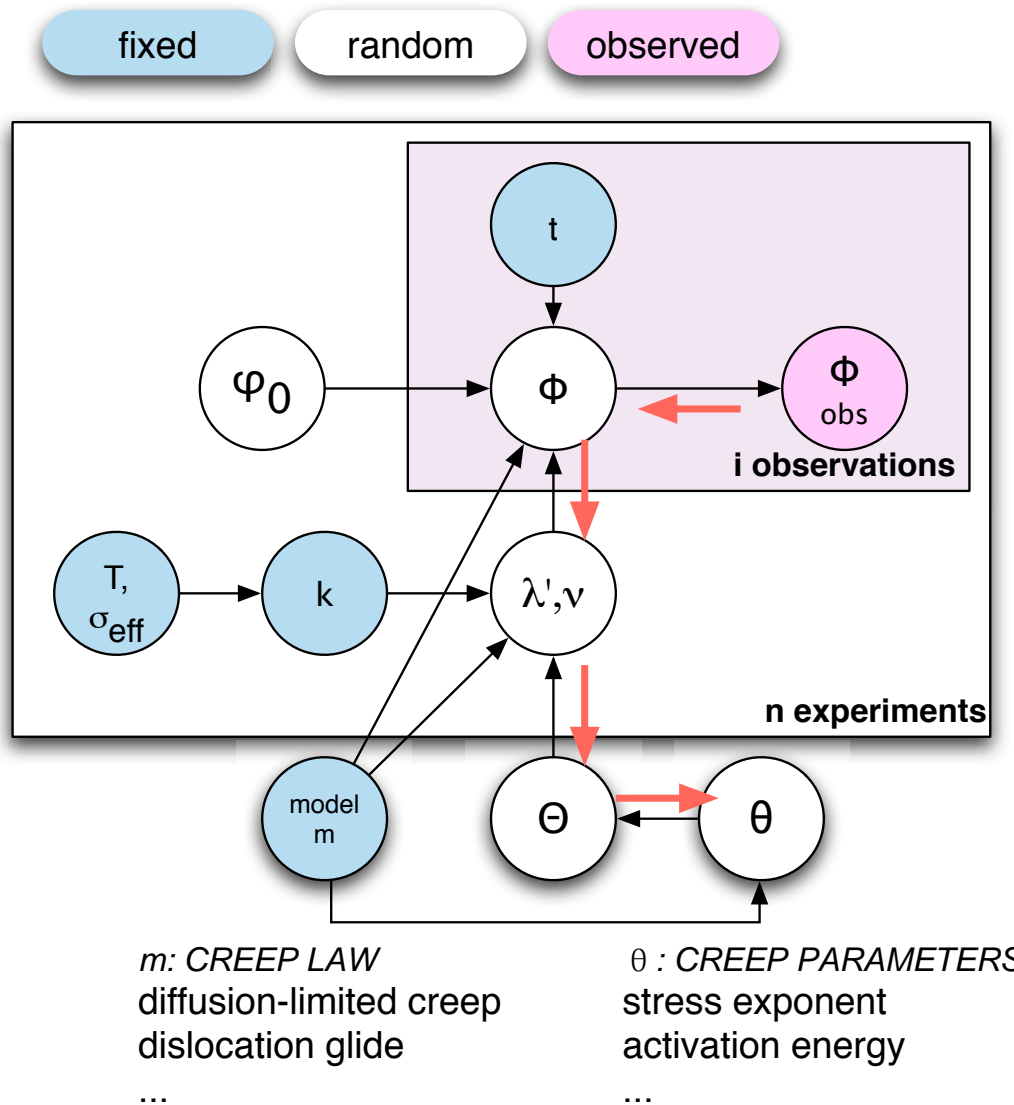
C: Revert from Θ to the creep parameters θ_i

E (Gaussian approx. of the posterior of θ)

$$P(\theta | \{\{\phi_{\text{obs}}^i, \varphi_{\text{obs}}^i\}^n\}) \simeq P(\theta | \{\lambda_{\text{obs}}^n, \nu_{\text{obs}}^n\}) \simeq G_{\theta}(F^{-1}(\hat{\Theta}), (J^T B J)^{-1})$$

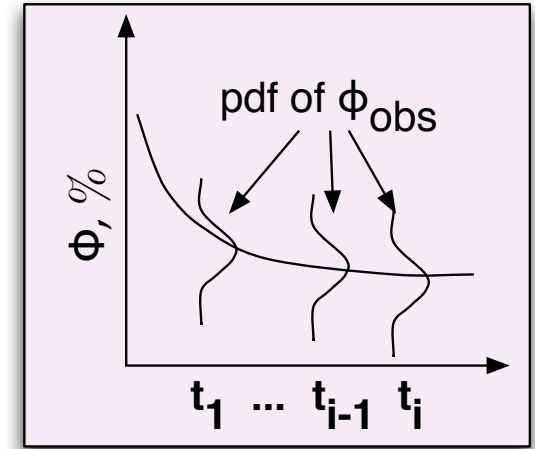
Hierarchical inference

"Elementary steps"



→ Generative model

Generate ϕ_{obs} for each experiment



↔ with $\Phi(t) = m(T, \sigma_{eff}, \theta, \varphi_0, t)$

← Inference

Find the pdf of θ using all the ϕ_{obs} , assuming that m reflects the dominant process during all of the experiments.

Identifying transitions

