Using the constructal law to predict climate change versus atmospheric properties

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Abstract — A simple and transparent model, based on the constructal law, is used to study climate change as a response to changes in atmospheric properties (albedo-ρ- and greenhouse factor –γ-). This model is first used to study the climate response to present changes in albedo and greenhouse factor. This allows comparing, in terms of energy imbalance and temperature, the climate change observed between 1880 and 2003 and that given by the model. Then, the model is used to predict future climate changes depending on several scenarios for the increase of atmospheric properties. This approach proves that the constructal law can help in defining new scenarios in relation to carbon emission policies. As an example, two scenarios are therefore tested and it is shown that the 2K critical temperature increase can be avoided only if a global low carbon energy pathway strategy is decided well before 2060.

Keywords — Constructal law, simplified model

1 INTRODUCTION

The constructal law governs the evolution of flow configuration in time for unsteady systems. It has been successfully applied in several fields. The constructal law has been first applied with success to predict pattern in nature in many domains other than global climate, for example, river basins scaling, the distribution of human settlements in geography, animal locomotion (running, flying, swimming), dendritic solidification, turbulent flow structure, human dynamics (e.g. urban design, city traffic), allometric scaling in biology, cracks in shrinking solids, dendritic aggregation of dust particles, heterogeneous multiscale porous media, etc.

In the case of climate change, the time evolution of flow configuration relies on the study of the heat current q which is driven from the high tropical temperature to the low polar temperature by the buoyancy effect in the layer of air that covers the Earth's surface. This literature was reviewed in Refs. [1-3].

In this paper, first, it is shown how the simple model used is able to account for the current thermal state of Earth. Then, the model is used to predict climate change in the future depending on changes in atmospheric properties.

2 MODEL

The earth surface (total area $A = 4\pi R^2$, where R is the radius) is modeled as shown in Fig. 1. The surface is divided into an equatorial zone of area A_H and temperature $T_H(t)$, and a polar zone of total area A_L and temperature $T_L(t)$. The first law of thermodynamics is applied to the equatorial and polar zones considered as closed systems.

The heat current between the equatorial and the polar zones is driven from T_H to T_L by the buoyancy effect in the layer of fluid that covers the Earth's surface.

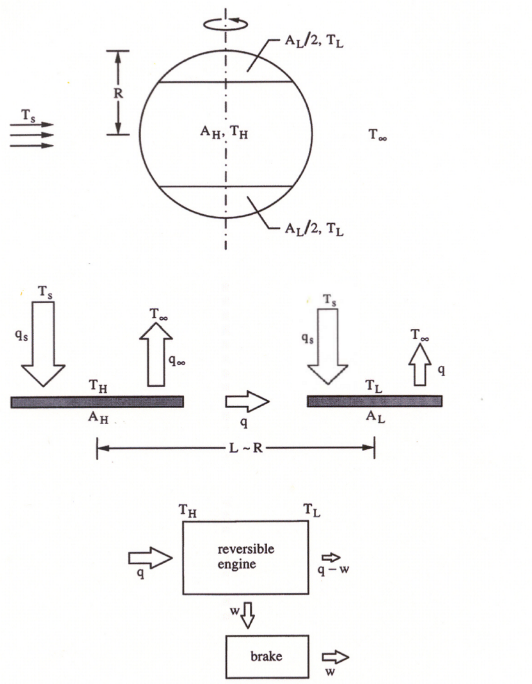

Fig. 1: Earth model with warm (A_H) and cold (A_L) zones, latitudinal convective heat current between them.

The equatorial and polar zones are treated as closed systems because the net mass flow affected by the counterflow is zero, and because the enthalpy current carried by the counterflow is analogous to a heat current across a surface with zero mass flow.

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The fluid layer covers an area of flow length L(~R) and width W(~R). The length L bridges the gap between TH and TL. More details on the model can be found in ref [4].

At the T_H-end of the fluid layer, the hydrostatic pressure at the bottom of the layer is $\rho_H gH$. Similarly, at the T_L-end, the pressure is $\rho_L gH$. The pressure difference in the L direction existing between the T_H-end and the T_L-end of the fluid layer is given by:

$$\Delta P \approx (\rho_L - \rho_H) gH \approx \rho\beta (T_H - T_L) \mathrm{g H} \qquad (1)$$

where ρ is the mean fluid density, β is the coefficient of volumetric thermal expansion and H is the vertical dimension of the fluid system that mixes (transfers momentum vertically) while moving horizontally.

The fluid-layer control volume is exposed to the force ΔPWH in the L direction. This force is opposed by the shear force felt by the moving fluid over the surface LW:

$$\Delta PWH = \tau LW \qquad (2)$$

The average shear stress is

$$\rho\varepsilon_M \frac{u}{H} \qquad (3)$$

where εM is the eddy diffusivity for momentum, and u is the velocity in the L direction. For the order of magnitude of εM we use Prandtl's mixing length model (e.g., Ref. [5], p. 343), in which we take H to represent the mixing length,

$$\varepsilon_M = H^2 \frac{u}{H} = Hu \qquad (4)$$

By eliminating ΔP, u and εM between Eqs. (1) - (4) we obtain the horizontal velocity scale

$$u \approx \left[\beta g (T_H - T_L) \frac{H^2}{L} \right]^{1/2} \qquad (5)$$

According to constructal theory, the configuration that will prevail is the one that provides a greater conductance for the flow of heat. It was shown in Ref. [1] that greater access is provided by the configuration when the loop is a vertical plane yielding:

$$q \approx \rho c_p (g\beta)^{1/2} H^2 R^{1/2} (T_H - T_L)^{3/2} \qquad (6)$$

for q, the rate of heat convection from the equatorial zone to the polar zone.

The dimensionless formulation of the governing equations is based on using the following scales

$$T_{scale} = f^{1/4} T_s = 392.8\,\mathrm{K} \qquad (7)$$

$$t_{scale} = \frac{(\rho_w hc)}{(\sigma T_{scale}^3)} \qquad (8)$$

where Ts, σ and f are the temperature of the Sun as a black body (5762 K), the Stefan Boltzmann constant (5.67 × 10–8 W m–2K–4), the Earth-Sun view factor (2.16 × 10–5). Using the dimensionless variables:

$$(\tilde{T}_H, \tilde{T}_L) = \frac{(T_H, T_L)}{T_{scale}} \qquad (9)$$

$$\tilde{t} = \frac{t}{t_{scale}} \qquad (10)$$

$$\tilde{q} = \frac{q}{(\sigma T_{scale}^4 A)} \qquad (11)$$

Equation (6) becomes [4]

$$x \left(\frac{d\tilde{T}_H}{d\tilde{t}} \right) = x f_H (1-\rho) - x(1-\gamma) \tilde{T}_H^4 - \tilde{q} \qquad (13) \qquad (12)$$

$$\tilde{q} \approx C (\tilde{T}_H - \tilde{T}_L)^{3/2} \qquad (14)$$

where x is the fraction of the body of water covering the equatorial zone, AH ($x = M_H/M$; M_H is the body of water covering A_H and M is the total ocean mass) and C is the group

$$C = \frac{(\rho c_p (g\beta)^{1/2} H^2 R^{1/2})}{(\sigma A T_{scale}^{5/2})} \qquad (15)$$

If we use H ~ 2 km for the thickness of the mixing layer in the atmosphere at 0 °C, R ~ 6600 km and A = 4πR2, then C ~ 0.04. On the other hand, the conductance C3/2 of Table 1 in ref [1] corresponds to assumption that H ~ 5.2 km, in which case C ~ 0.181. The numerical results reported in the next section are based on C = 0.181.

In summary, there are three equations [Eqs. (12) - (14)] containing four unknowns, which are functions of time. The radiative parameters are assumed specified. The problem is closed by invoking the constructal law, which states that the design of the flow system evolves toward greater flow access for the heat current from the hot zone to the cold zone,

$$\partial\tilde{q} / \partial x = 0 \qquad (16)$$

The evolution toward greater flow access is achieved by selecting the configuration, which is represented by x.

3 Results

To start with, as reference configuration we generated the steady state that prevails when ρ = 0.3 and γ = 0.4. The results of this optimization are

$$\begin{aligned} &\rho = 0.3 \quad x = 0.8401 \\ &\gamma = 0.4 \quad \tilde{T}_H = 0.7471 \, (T_H = 293.5\,K) \\ &\qquad\quad \tilde{T}_L = 0.6577 \, (T_L = 258.4\,\mathrm{K}) \end{aligned} \qquad (17)$$

The average surface temperature = 293.5 × 0.8401 + 258.4 × (1 – 0.8401) = 288.3 K (15.2 °C) which is close to the actual value (~ 18 °C).

3.1 Step change in albedo and greenhouse factor

The first atmospheric properties modifications studied were step changes starting from the reference configuration.

Two step changes in the albedo and the greenhouse factor are considered: (A) $\delta\rho = 0.002$ with $\delta\gamma = 0.011$, and (A1) $\delta\rho = 0.002$ with $\delta\gamma = 0.005$ (Fig. 2). Four time scales are plotted: one for the non-dimensional time and three others corresponding to different ratio of ocean layer mass taken into account for Earth's inertia (case 1: 100 %, case 2: 50 % and case 3: 33 %).

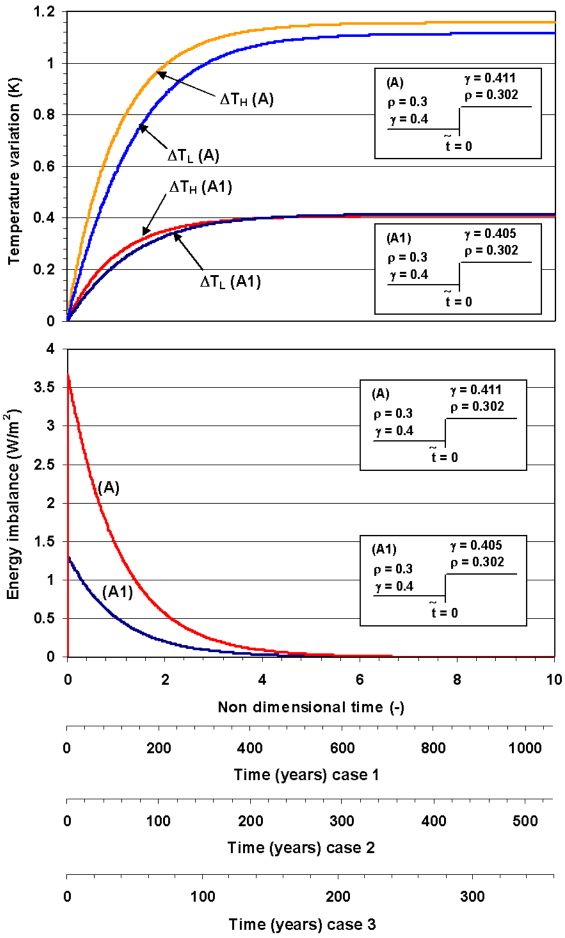

Fig. 2: Earth's energy flux imbalance and temperature increase for the polar zone (ΔT_L) and the equatorial zone (ΔT_H) evolutions in response to two different step changes in albedo (ρ) and greenhouse factor (γ). Four time scales are plotted: non dimensional time, and three cases corresponding to different ratio of ocean layer mass taken into account for Earth's inertia (case 1: 100 %, case 2: 50 % and case 3: 33 %)

For case (A) the new steady state calculated based on the model is represented by:

$$\rho = 0.302 \qquad x = 0.8398$$
$$\gamma = 0.411 \qquad (18)$$

while for case (A1), it is represented by:

$$\rho = 0.302 \qquad x = 0.8395$$
$$\gamma = 0.405 \qquad (19)$$

The evolution of the Earth's energy imbalance is also reported in Fig. 2. The energy imbalance is defined as the net heat current received from the sun minus the net heat current rejected to the outer space :

$$q''_{ex} = \frac{1}{A}\left[(q_{sH} + q_{sL}) - (q_{H\infty} + q_{L\infty})\right] \qquad (20)$$

The step change creates an energy imbalance at the Earth's surface and, at constant atmospheric properties, the only mean for this energy imbalance to be compensated is an Earth's temperature increase. However, due to Earth's thermal inertia, it takes a long time to reach the new equilibrium when the energy imbalance vanishes. In case A, a 3.67W/m² (1.3 W/m² in case A1) energy imbalance results in an earth's overall mean temperature change equal to 1.15 K (0.41 K for case (A1)). The time needed for Earth's mean temperature to reach 60 percent of its equilibrium response is different for case (A) and for case (A1). Depending on the mass fraction of ocean layer taken into account in the inertia calculation, these times correspond to 104, 57 and 35 years for cases 1, 2 and 3, respectively in case A.

Although coming from a simple model, these results are consistent with those based on complex meteorological models. For example, using the global climate model of the NASA Goddard Institute of Space Studies to simulate the climate evolution for the 1880-2003 period, Hansen et al. [6] have found an overall temperature increase of 1.2 K, an energy imbalance of 0.85 ± 0.15 W/m2 after 120 years for a total energy imbalance of 1.8 W/m2 relative to 1880. Furthermore, 25-50 years are needed for the Earth's temperature to reach 60 % of its equilibrium response. Hansen et al. also reported that 85 % of the heat storage occurs above 750 m depth. The depth taken into account for thermal inertia calculation in case 3 (917 m) is of the same order of magnitude, so that only case 3 is studied in what follows.

The lessons to be drawn from the step changes results is that when an excess of energy does exist at the Earth's surface, a temperature increase will occur to reach a new equilibrium to cancel this excess of energy even though the atmospheric parameters are constant. That means that measures to mitigate global warming have to be taken much in advance to control the climate.

3.2 Ramp function for ρ and γ increase

Now, instead of a step change, a linear increase from the initial pre-industrial state to the present one of ρ and γ lasting 120 years is assumed to study the evolution due to the atmospheric properties but we assume that after that 120 years linear increase, the atmospheric parameters are constant. The initial and final values of ρ and γ are maintained the same as those used in the previous step changes. The x value is optimised at every time step during the evolution of ρ and γ.

The temperature and energy imbalance evolutions are presented on Fig. 3 by assuming that the Earth's inertia is equal to one third of the ocean mass.

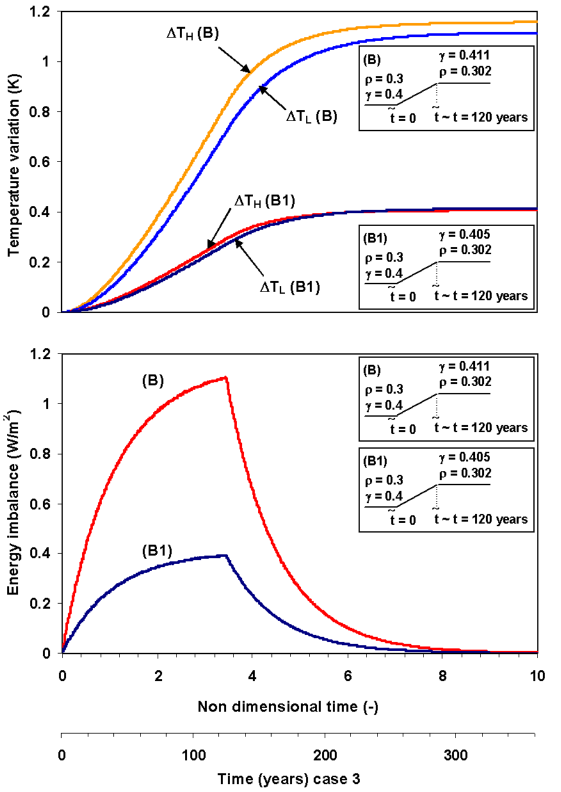

Fig. 3 Earth's energy flux imbalance and temperature increase for the polar zone (ΔT_L) and the equatorial zone (ΔT_H) evolutions in response to two different ramp changes in albedo (ρ) and greenhouse factor (γ).

Two ramp functions are used. In both cases we obtain the corresponding Earth's temperatures. As would be expected, the final temperatures reached in both cases are the same as those observed after step changes yielding the same final atmospheric properties (Figure 2) but the history to get those final temperatures differ greatly. With the ramp, the delay is extended.

As the values of γ and ρ were kept constant after 120 years, the results obtained after that date could be compared to those found by Wigley [7] for his constant concentration (CC) scenario. In his case, the reference year is year 2000, which in our case corresponds to an elapsed time of 120 years.

If we compare the results at year 2200 (i.e., an elapsed time of 320 years in our case), our calculations match quite well Wigley's results for the CC scenario: case (A) is close to his central climate sensitivity scenario (0.36 K compared to 0.33 K, respectively) while case (B) is close to the low sensitivity scenario (0.13 K compared to 0.15 K).

3.3 Continuous increases in ρ and γ

In this section, we present the results for a continuous linear increase of γ and ρ. The rates are equal to those used in the previous section, case (B). This model gives an idea of the Earth's response to a "business as usual" scenario. Two scenarios are considered:

- case (C), the rates of increase for γ and ρ are equal to their respective values used in section 1.2:
$$\rho = 0.3 + \frac{0.002 \cdot \tilde{t}}{3.45} \quad \text{and} \quad \gamma = 0.4 + \frac{0.011 \cdot \tilde{t}}{3.45}$$

This represents an optimistic view for γ, as it is known that the CO_2 emission rate has greatly increased recently. The CO_2 emissions growth rate was respectively 1% and 2.5 % per year during the periods 1990-1999 and 2000-2009 [8]. To improve our model, a change in the rate for γ (doubling), starting at year 2000, is implemented and corresponds to case (B):

- case (D) is identical to case (A) for $\tilde{t} \le 3.45$ (~ 120 years), and above this value the γ increase rate is multiplied by a factor of two:
$$\rho = 0.3 + \frac{0.002 \cdot \tilde{t}}{3.45} \quad \text{and} \quad \gamma = 0.4 + \frac{0.022 \cdot (\tilde{t} - 3.45)}{3.45}$$

for $\tilde{t} > 3.45$

The origin of time still corresponds to the year 1880, as it was the case in the previous section. Figure 4 shows that after 300 years the mean temperature increase is roughly 2.4 K for case (C) and 4.2 K for case (D).

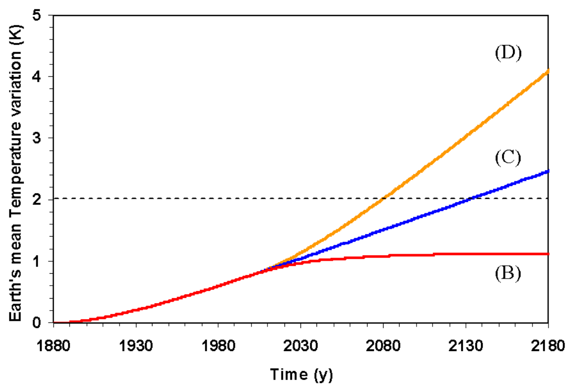

Fig. 4: Evolution of the Earth's mean temperature in response to different changes in albedo (ρ) and greenhouse factor (γ): (B) ramp with stabilization in year 2000, (C) continuous increase, (D) continuous increase with doubling of γ increase ratio at t = 2000.

For the period between 120 and 320 years, this corresponds to an increase of 1.88 K and 3.75 K for cases (C) and (D), respectively, values which correspond to the lower range of the temperature increase as predicted by IPCC models which predict from 1 to 4K temperature increase in 2100. Comparing these last results with the values found by Wigley [7] in his constant emissions (CE) scenario at year 2200, case (D) is slighlty above the Wigley's value for high climate sensitivity to CO_2 doubling (3.3 K) while case (C) is almost equal to the value found by Wigley for medium climate sensitivity (2.1 K).

Another reading of Fig. 4, is to evaluate the remaining time before reaching a temperature increase of 2 K, value above which, the consequences on climate are seen to be irreversible. This threshold is reached in 2080 and 2130 for cases (D) and (C), respectively. Hence, measured from today, there remain only 70 years in the worst case before reaching the 2 K increase. Lowering the CO_2 emissions to a rate close to that between years 1880 and 2000, would result in a 50 years delay.

Concerning the energy imbalance, it is evaluated by Wigley at 3.7 W/m2 for a CO_2 doubling. The values found based on our model 1.2 W/m2 and 2.7 W/m2 (Figure 5) remain below those of Wigley.

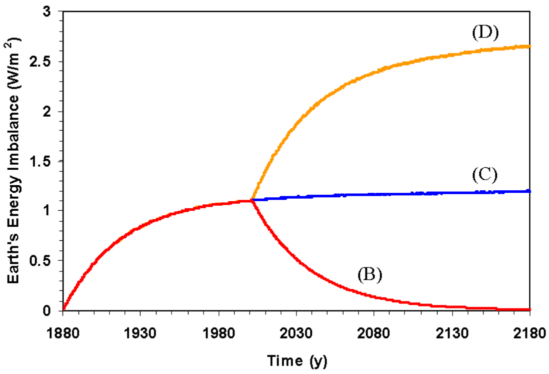

Fig. 5: Evolution of Earth's energy imbalance in response to different changes in albedo (ρ) and greenhouse factor (γ): (B) ramp with stabilization in year 2000, (C) continuous increase, (D) continuous increase with doubling of γ increase ratio at t = 2000.

However, the comparison between these values is not straightforward, as Wigley's value is achieved for CO_2 concentration doubling whereas in our case the energy imbalance depends on ρ and γ evolutions.

On Figure 4, the ramp simulation (curve B) shows that if the greenhouse factor would be controlled at its 2003 level, then the Earth's temperature should not approach the critical 2K value. But, this case does not correspond to the actual situation since GHG emissions are not presently severely reduced so as to get a constant greenhouse factor.

If the atmospheric properties increase, in the future, at the same rhythm than that observed up to now, the temperature raise will overpass the 2K critical value in 2130 but if the rhythm is doubled, then the 2K critical temperature raise will be reached in 2080. Moreover, in both cases, the energy imbalance increases dangerously (Figure 5, curves C and D), that means that even if political decisions were taken so as to control severely the atmospheric properties which should not increase any more, the temperature should still follow increasing due to Earth's thermal inertia. Decisions to reduce GHG emissions have to be taken a long time before the Earth's temperature increase approaches 2K.

3.4 Voluntary strategy to reduce emissions

Noting that the present global strategy on GHG emissions yields an unacceptable temperature increase, two new scenarios based on a voluntary GHG emissions reduction are looked at. On Figure 6 is presented the response of the Earth's temperature variation in 3 cases. The first scenario corresponds to the continuous increase in ρ and γ (curve D) just discussed previously and two new scenarios (E and F). Scenario F assumes that drastic decisions are taken so as to reduce considerably the GHG emissions and to reach constant atmospheric parameters by 2030 whereas scenario E assumes the constant atmospheric parameters to be reached by 2060. In both new scenarios when constant atmospheric parameters are achieved, the temperature reaches a plateau (Figure 6) but due to the high energy imbalance (Figure 7), the temperature follows increasing after the constant atmospheric parameters is reached.

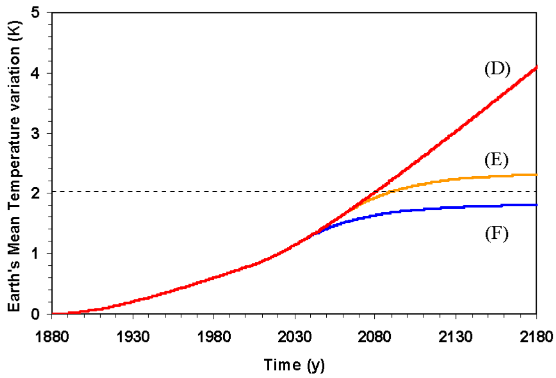

Fig. 6: Evolution of Earth's mean temperature in response to a continuous change (constant emission scenario) in albedo (ρ) and greenhouse factor (γ) for three scenarios: (D) doubling of γ increase ratio at t = 2000, (E) doubling of γ increase ratio at t = 2000 and stabilization of γ and ρ at t = 2030, (F) equivalent to case (E) with stabilization at t = 2060.

The total temperature raise does not reach 2K when the emissions are stopped in 2030 as looked at but it overpasses the critical 2K value when the emissions last until 2060.

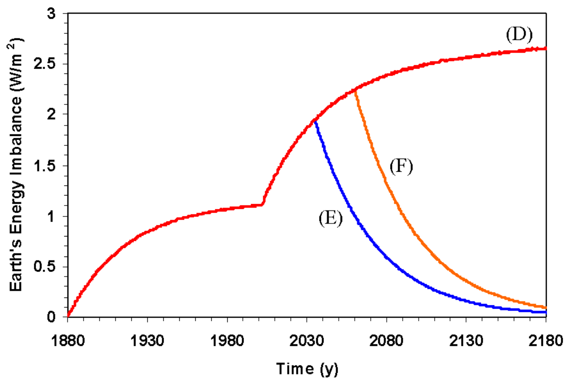

Fig. 7: Evolution of Earth's energy imbalance in response to a continuous change (constant emission scenario) in albedo (ρ) and greenhouse factor (γ) for three scenarios: (D) doubling of γ increase ratio at year 2000, (E) doubling of γ increase ratio at year 2000 and stabilization of γ and ρ at year 2030, (F) equivalent to case (E) with stabilization at year 2060.

From Figure 7, we note a high Earth's energy imbalance which takes more than 100 years to be completely resorbed and explains the temperature variation after the time when the atmospheric parameters are constant. That proves the need for a urgent global strategy to mitigate global warming.

From Figure 6, we can conclude that, to avoid a temperature increase which could reach 4K or even more in 2180, the challenge is to reach a constant GHG concentration pathway so as to get constant atmospheric parameters around 2030. If such a pathway is not achieved before 2060, the risk is very high to loose control on the climate.

4 CONCLUSION

The question on how to achieve this energy transition within 30 years from an energy society intensive in carbon emission to a society at constant carbon concentration is open. Although it is not the aim of that paper, we can say that, fortunately, routes do exist: energy saving, decarbonated energy sources, CO_2 capture and sequestration, etc. But, some worrying signals do exist presently like the development of unconventional fossil fuels, including shale gas. Many policymakers view the abundance of affordable shale gas as creating an unprecedented opportunity to use natural gas as a bridge fuel to a 21st-century energy economy that relies on efficiency, renewable sources, and low-carbon fossil fuels such as natural gas. However, without CO_2 capture and sequestration, it will add more carbon into the atmosphere and it represents a strong threat for long term unacceptable GHG emissions and Earth's temperature amplification.

The model discussed herein, based on the constructal law, can be an appropriate tool, thanks to its simplicity, to check GHG scenarios. Work is under progress to improve the model by the introduction of correlations between the atmospheric properties and the GHG emissions and concentration.

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